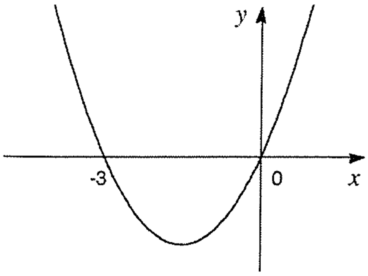
Section One: Calculator Free **(24 marks)**

This section has **five (5)** questions. Answer **all** questions. Write your answers in the spaces provided.

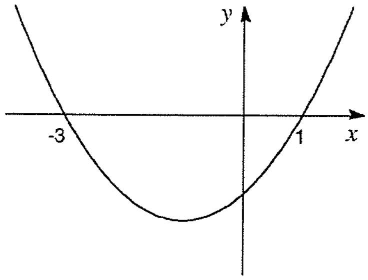
Working time for this section is 25 minutes.

Question 1 **(3 marks)**

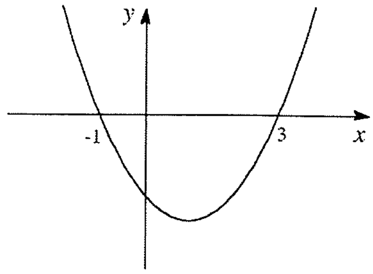
Shown are eight quadratic functions, numbered 1 to 8, and five graphs, lettered A to E.
Each graph corresponds to one of the functions. Decide which function goes with which graph.
You will have three functions left over.



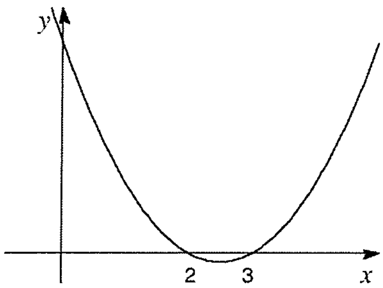
A 1



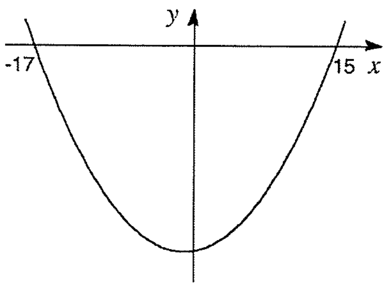
B 2



C 7



D 6



E 3

$y = x^2 + 3x$

$y = (x-1)(x+3)$

$y = (x+17)(x-15)$

$y = (x+2)(x+3)$

$y = (x-17)(x+15)$

$y = x^2 - 5x + 6$

$y = (x+1)(x-3)$

$y = x(x-3)$

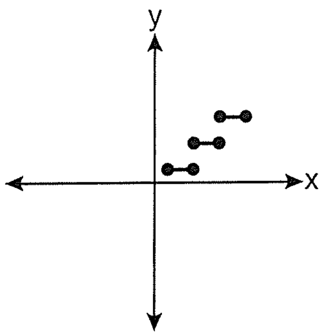
Solution	Specific behaviours	Point
$A\ y = x^2 + 3x$ $B\ y = (x-1)(x+3)$ $C\ y = (x+1)(x-3)$ $D\ y = x^2 - 5x + 6$ $E\ y = (x+17)(x-15)$	✓ Identifies two quadratics. ✓ Identifies four quadratics ✓ Identifies all five.	?

Question 2 **(5 marks)**

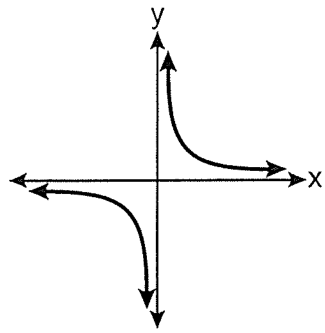
Circle the correct answer for the following questions:

(a) Which represents a relation that is not a function? (1 mark)

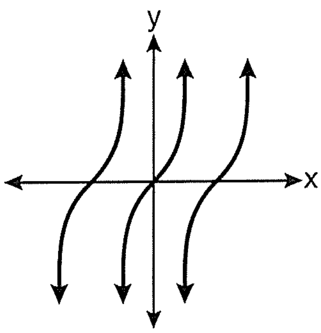
A



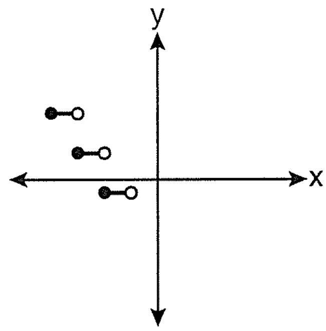
B



C



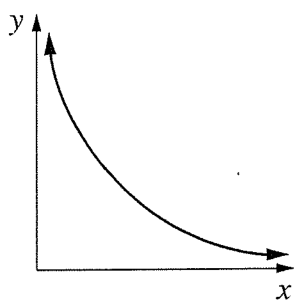
D



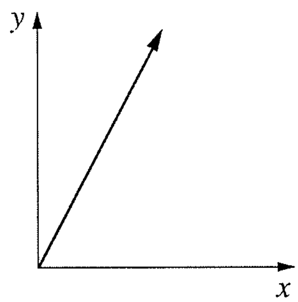
Solution	Specific behaviours
A	✓ Correct answer.

(b) Which graph shows that y is directly proportional to x? (1 mark)

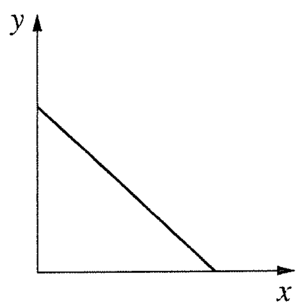
A



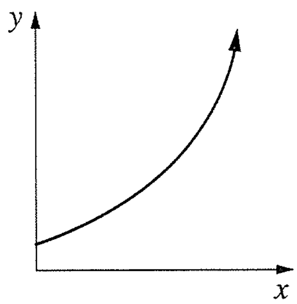
B



C



D

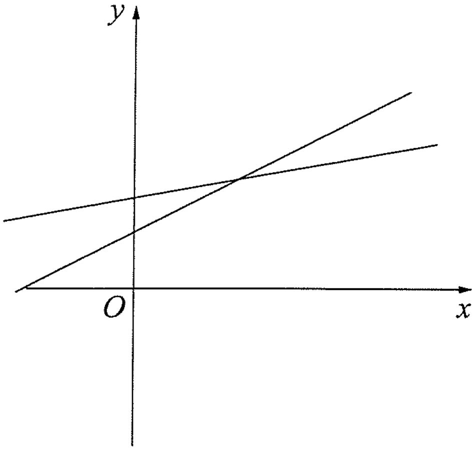


Solution	Specific behaviours
B	✓ Correct answer.

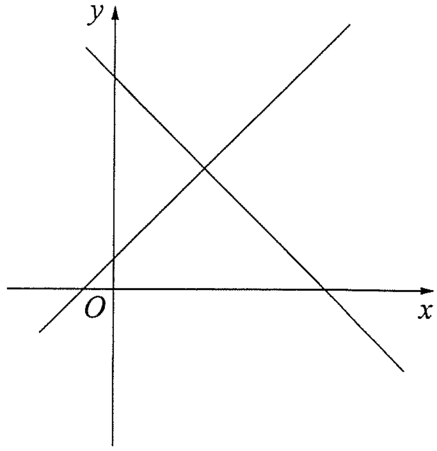
(c) George drew a correct diagram that gave the solution to the simultaneous equations $y = 2x - 5$ and $y = x + 6$.

Which diagram did he draw? (1 mark)

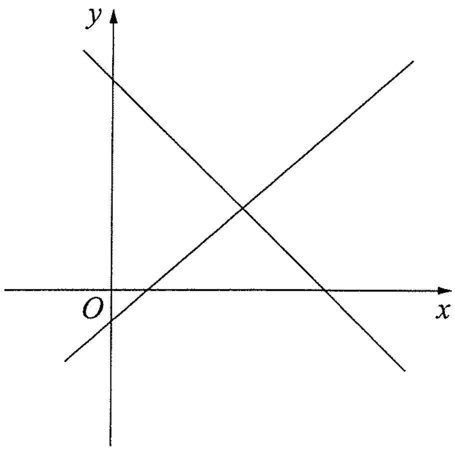
A



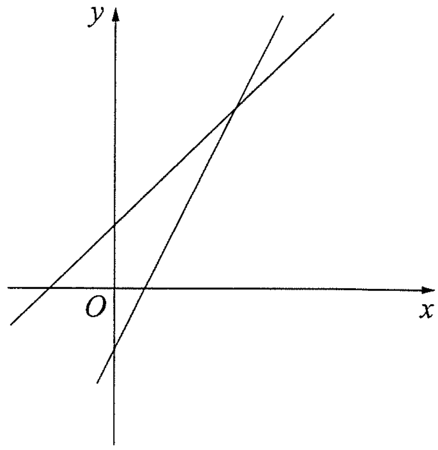
B



C



D



Solution	Specific behaviours
D	✓ Correct answer.

(d) Which equation represents the relationship between x and y in this table? (1 mark)

x	0	2	4	6	8
y	1	2	3	4	5

A $y = 2x + 1$

B $y = 2x - 2$

C $y = \frac{x}{2} - 2$

D $y = \frac{x}{2} + 1$

Solution	Specific behaviours
D	✓ Correct answer.

(e) For the graph $y = ax^2 + bx + c$, if a and c are both positive, which of the following statements is true. (1 mark)

A The graph will have a minimum turning point, and a positive y-intercept.

B The graph will have a maximum turning point, and a positive y-intercept.

C The graph will have a maximum turning point, and has two positive x intercepts.

D The graph will have a minimum turning point, and a negative y-intercept.

Solution	Specific behaviours
A	✓ Correct answer.

Question 3 **(2 marks)**

The linear function $f(x) = 4 - x$ has range $-2 \le f(x) < 6$.

Determine the domain of the function.

Solution	Specific behaviours
$-2 = 4 - x$ $x = 6$ $6 = 4 - x$ $x = -2$ $D_x : \{x : -2 < x \le 6\}$	✓ Works backwards from range to determine values for domain. ✓ States domain.

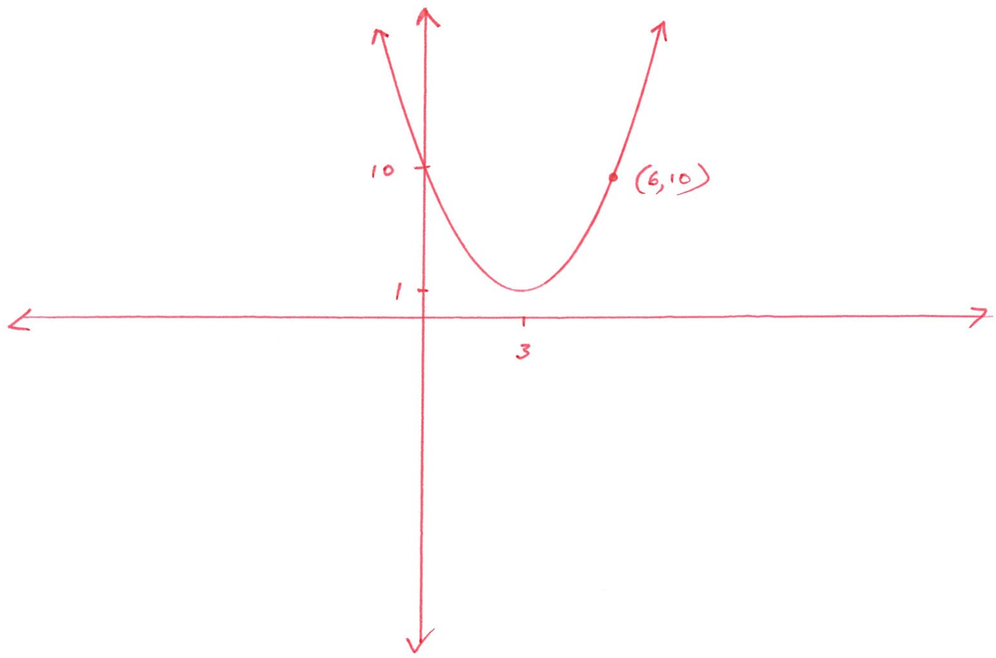
Question 4 **(6 marks)**

(a) Complete the square for $x^2-6x+10$. (2 marks)

Solution	Specific behaviours	Point
$x^2-6x+10=(x-3)^2-9+10$ $=(x-3)^2+1$	✓ Uses completing the square. ✓ Correct answer.	?

(b) Using your result from part (a), sketch the graph $y=x^2-6x+10$, showing all significant features. (3 marks)

Solution	Specific behaviours	Point
	✓ Turning point at (3, 1). ✓ y-intercept at (0, 10). ✓ Correct graph.	?



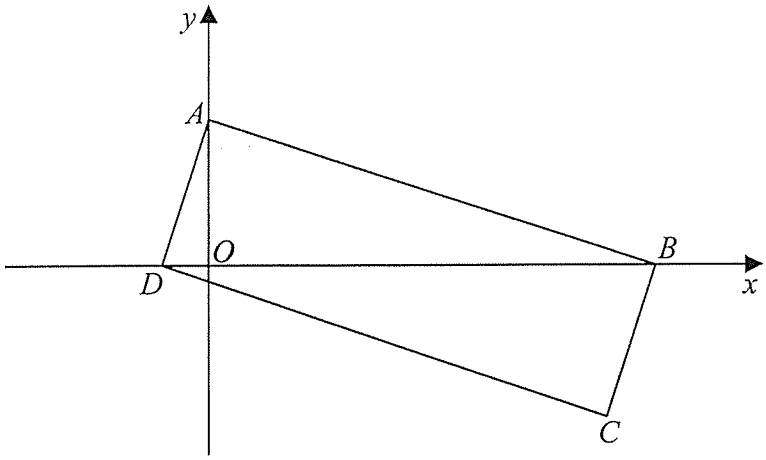
(c) Explain how the graph can be used to show the following statement is always true: (1 mark)

"When a real value of x is substituted into $x^2-6x+10$ the result is positive."

Solution	Specific behaviours	Point
Since the curve is always above the x-axis, then $y=x^2-6x+10$ would always been positive.	✓ Recognises that curve is always above x-axis, and hence answer is always positive.	?

Question 5 **(8 marks)**

The figure below shows a rectangle $ABCD$.



The point A lies on the y-axis and the points B and D lie on the x-axis as shown.

Given that the straight line through the points A and B has equation $5y+2x=10$

(a) show that the straight line through the points A and D has equation $2y-5x=4$. (3 marks)

Solution	Specific behaviours	Point
$AB: y=-\frac{2}{5}x+2 \Rightarrow m_{\perp}=\frac{5}{2}$ $AD: y=\frac{5}{2}x+2$ $2y=5x+4$ $2y-5x=4$	✓ Determines perpendicular gradient. ✓ Determines equation of AD. ✓ Rearranges.	1.1.4 1.1.5

(b) determine the coordinates of the points B and D. (2 marks)

Solution	Specific behaviours	Point
$2x=10$ $x=5 \Rightarrow B(5,0)$ $-5x=4$ $x=-\frac{4}{5} \Rightarrow D\left(-\frac{4}{5},0\right)$	✓ Determines coordinates of B. ✓ Determines coordinates of D.	1.1.5 10A

(c) determine the coordinates of the midpoint of the diagonal BD. (1 mark)

Solution	Specific behaviours	Point
$\dfrac{-\frac{4}{5}+5}{2}=\dfrac{21}{10}\quad i.e.\left(\dfrac{21}{10},0\right)$	✓ Determines coordinates of midpoint.	?

(d) The diagonals of a rectangle bisect. *Use this fact*, along with your results from part (a) and (c), to determine the coordinates of the point C. (2 marks)

Solution	Specific behaviours	Point
$\left(\dfrac{0+x}{2},\dfrac{2+y}{2}\right)=\left(\dfrac{21}{10},0\right)=(2.1,0)$ $C\left(\dfrac{21}{5},-2\right)=(4.2,-2)$	✓ Determines midpoint formula, or 'steps it out'. ✓ Determines coordinates.	?

End of Calculator Free Section

Section Two: Calculator Assumed **(31 marks)**

This section has **four (4)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 30 minutes.

Question 6 **(6 marks)**

An air balloon leaves its base at 12 noon and is moving such that its height, h metres, above sea-level at any time t hours, after 12 noon, is given as

$$h(t) = -\frac{3}{4}(t+1)(t-16) \quad \text{for } 0 \le t \le 20$$

Determine:

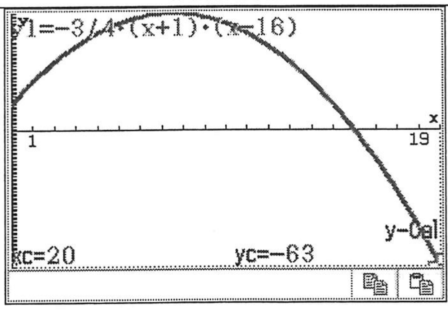
(a) the initial height above sea-level. (1 mark)

Solution	Specific behaviours
$t=0$ $h(0)$ = 12 m	✓ Correct answer.

(b) the maximum height, correct to one decimal place, to which the balloon rises. (1 mark)

Solution	Specific behaviours
$h(7.5)$ = 54.2 m	✓ Correct answer.

(c) the minimum height to which the balloon sinks over the time interval. (2 marks)

Solution	Specific behaviours
 When $t = 20$, $h = -63$ i.e. min height is 63 m below sea level	✓ Determines minimum occurs at (20, -63). ✓ Correctly interprets in the context of the question.

The balloon is to manoeuvre over a building of height 30 metres above sea level.

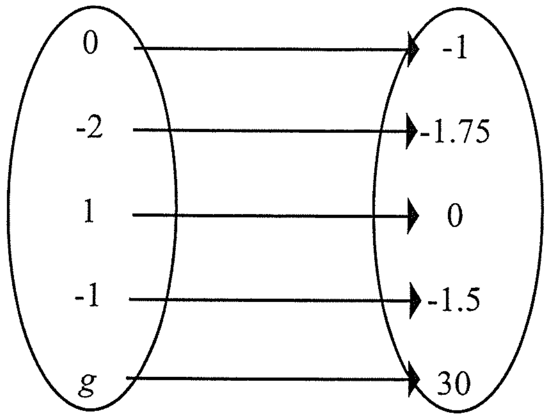
(d) During what times, correct to two decimal places, will it be able to do this? (2 marks)

Solution	Specific behaviours
$30 = -\frac{3}{4}(t+1)(t-16)$ $1.82 \le t \le 13.18$	✓ Writes equation, or indicates on graph solution method. ✓ States times.

ie between 1:49pm & 1:11 am the next day.

Question 7 **(7 marks)**

The mapping below is of the form $f: x \rightarrow a \times 2^x + b$ and maps the elements of x to elements of y.



(a) List the elements in the domain of $f(x)$. (1 mark)

Solution	Specific behaviours
$D_x = \{-2, -1, 0, 1, g\}$	✓ Determines the domain.

(b) List the elements in the range of $f(x)$. (1 mark)

Solution	Specific behaviours
$R_y = \{-1.75, -1.5, -1, 0, 30\}$	✓ Determines the range.

(c) Find a and b. (3 marks)

Solution	Specific behaviours
$(0,-1) \Rightarrow -1 = a + b \quad (1)$ $(1,0) \Rightarrow 0 = 2a + b \quad (2)$ $(2)-(1) \Rightarrow 1 = a$ $\Rightarrow b = -2$	✓ Sets up simultaneous equations. ✓ Correct value for a. ✓ Correct value for b.

(d) Find the value of g. (2 marks)

Solution	Specific behaviours
$f(x) = 2^x - 2$ $30 = 2^g - 2$ $32 = 2^g$ $g = 5$	✓ Sets up equation. ✓ Correct value for g.

Question 8 **(9 marks)**

(a) Show that the lines $y+2x=3$ and $2y-x=1$ are perpendicular. At what point do they intersect? (3 marks)

Solution	Specific behaviours	Point
$y=-2x+3$		?
$y=\frac{1}{2}x+\frac{1}{2}$	✓ Rearranges.	
$\frac{1}{2}\times(-2)=-1\Rightarrow$ perpendicular	✓ Shows they are perpendicular.	
$\left.\begin{array}{l} y+2x=3 \\ 2y-x=1 \end{array}\right\}(1,1)$	✓ Determines point of intersection.	

(b) Determine the equation of the line, having an x-intercept of –4, and which is parallel to the line connecting the turning point of $y=(x+1)^2+3$ with (3, 7). (3 marks)

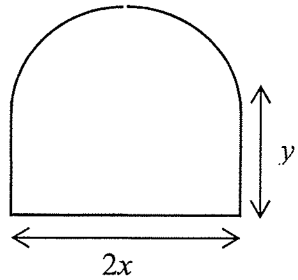
Solution	Specific behaviours	Point
$TP(-1,3)$	✓ Determines turning point of parabola.	?
$m=\frac{7-3}{3-(-1)}$		
$m=1$	✓ Determines gradient of line through turning point and (3, 7).	
$y=x+c$		
$0=-4+c$		
$c=4$		
$y=x+4$	✓ Determines equation of line.	

(c) The line with an angle of inclination to the positive x axis of 135° and y intercept of $-\frac{1}{2}$.
State your answer in the form $ax+by=c$, where a, b and c are integers. (3 marks)

Solution	Specific behaviours	Point
$m=\tan 135°=-1$	✓ Determines gradient.	?
$y=-x-\frac{1}{2}$	✓ Determines equation of the line.	
$x+y=-\frac{1}{2}$		
$2x+2y=-1$	✓ Rearranges into required form.	

Question 9 **(9 marks)**

A window pane is to be made from 12 metres of steel.
The pane is to have a rectangular base and a semi-circular top as shown.



If the base of the pane is $2x$ metres and the side y metres then:

(a) show that $y = 6 - x - \frac{\pi}{2}x$. (2 marks)

Solution	Specific behaviours	Point
$12 = 2x + 2y + \frac{1}{2} \times 2\pi x$ $12 = 2x + \pi x + 2y$ $2y = 12 - 2x - \pi x$ $y = 6 - x - \frac{\pi}{2}x$	✓ Works out equation for perimeter. ✓ Rearranges into required form.	?

(b) show that the area enclosed by the pane is given by $A(x) = 12x - 2x^2 - \frac{\pi}{2}x^2$. (3 marks)

Solution	Specific behaviours	Point
$A(x) = 2xy + \frac{1}{2}\pi x^2$ $A(x) = 2x\left(6 - x - \frac{\pi}{2}x\right) + \frac{\pi}{2}x^2$ $A(x) = 12x - 2x^2 - \pi x^2 + \frac{\pi}{2}x^2$ $A(x) = 12x - 2x^2 - \frac{\pi}{2}x^2$	✓ Works out equation for area. ✓ Substitutes in answer from part $y = 6 - x - \frac{\pi}{2}x$. ✓ Expands and shows required formula.	?

(c) calculate the maximum area enclosed by the pane, and the length of the base and side that gives the maximum area. State all answers correct to 1 decimal place. (4 marks)

Solution	Specific behaviours	Point
max TP at $(1.68, 10.08)$ Maximum area is $10.1\ \text{m}^2$ Base length is 3.4 m Side length is 1.7 m	✓ Determines maximum TP. ✓ States maximum area. ✓ States base length. ✓ States side length.	?

End of Calculator Assumed Section